

ELASTOPLASTIC DEFORMATION OF BODIES INTERACTING THROUGH CONTACT UNDER THE ACTION OF PULSED ELECTROMAGNETIC FIELD

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A formulation is presented for the problem of elastoplastic deformation of the system of electrically conductive bodies under the action of pulsed electromagnetic field. The numerical technique for solving the problem is based on the finite element method. Deformation of bodies interacting through contact during the pulsed magnetic treatment of materials is analyzed. The influence of the manufacturing and design parameters of the inductor–workpiece system on its stress-strain state is investigated.

Keywords: elastoplastic deformation, finite element method, pulsed magnetic treatment.

Introduction. A prerequisite for the functioning of many industrial bodies and bodies interacting through contact is the presence of the electromagnetic field (EMF). Its interaction with electrically conductive bodies results in their motion or deformation. In this case, the levels of the EMF energy may be so high as to cause irreversible deformation or fracture of structural elements. For manufacturing purposes, materials are produced, treated with pulsed EMF in order to increase their strength and corrosion resistance and to reduce their residual stress level [1–4]. Many production processes of pressure treatment involve the EMF energy. A review of advances in the technologies of pulsed magnetic treatment of materials (PMTM) and the state of the art of the problem of simulation, design and development of manufacturing processes and the equipment for this kind of treatment are presented in [5].

Noteworthy is that a sufficient number of studies are currently known devoted to the simulation of the plastic deformation of workpieces with the PMTM based on the analysis of their stress-strain state (SSS). The simulation results for the process of the electromagnetic pulse forming of thin-walled workpieces using a continuum thermodynamic model are reported in [6], with the constitutive relations derived for the electromagnetic field components. The workpiece deformation is described using an anisotropic viscoelastoplastic material. The coupling between the EMF and the stress and strain fields is established by the electromagnetic forces in the equations of motion. The results for the elastoplastic deformation of a thin-walled aluminum cylinder are reported as an example.

The results for a numerical simulation of electromagnetic sheet bulging using pulsed electromagnetic fields are given in [7]. The simulation is performed by the finite element method and consists of two steps: simulation of the propagation of electromagnetic fields in the workpiece, followed by its elastoplastic deformation. The finite element model includes the workpiece, a flat multi-turn inductor, or coil, and ambient medium (air). The space-time distributions of the electromagnetic field vector components and deformation tensor components are presented. The numerical analysis of the distribution of electromagnetic field and electromagnetic force components in the electromagnetic forming of sheet metals is performed in [8]. The problem was solved in the axisymmetrical formulation using the developed FE-model for the flat multi-turn coil, workpiece and ambient medium. The space-time distributions of the electromagnetic forces acting on the workpiece are presented. The special features of the external EMF are established, implying that the forces of attraction to the coil prevail over those of repulsion.

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At the same time, no sufficient attention is paid to the SSS analysis of the electromagnetic field sources, such as coils. It is known that during the current pulse generation, strong electromagnetic forces, significant in magnitude, act on the coil, which may result in its irreversible deformation. With the variation of the coil shape, the spatial form of the generated pulse is distorted, which adversely affects the production process. The SSS analysis of the coil allows the formulation of recommendations for its design. Therefore, the development of the efficient methods for the SSS analysis of electrically conductive compound bodies and the strength evaluation are urgent from the scientific and practical standpoint.

Mathematical Problem Formulation. Consider a general formulation of the problem of elastoplastic deformation of a system of bodies interacting through contact in the presence of EMF. Let a system of bodies be given in Cartesian coordinates x_i , $i=1, 2, 3$. The body of volume V_j has the surface S_j ($S_j = S_{jp} \cup S_{ju} \cup S_{jc}$), where S_{jp} , S_{ju} , and S_{jc} are the body parts for which the external distributed forces and conditions of fastening and contact interaction are specified.

The electromagnetic processes in the absence of free charges are described by the following set of fundamental Maxwell's equations:

$$\text{rot } \vec{H} = \varepsilon_c \frac{\partial \vec{E}}{\partial t} + \vec{j}, \quad \text{rot } \vec{E} = -\mu_c \frac{\partial \vec{H}}{\partial t}, \quad \text{div } \vec{H} = 0, \quad \text{div } \vec{E} = 0, \quad (1)$$

where \vec{j} , \vec{E} , and \vec{H} are the current density, electric field and magnetic field intensities, respectively, and μ_c and ε_c are the magnetic permeability and electric permittivity of the material.

With the neglect of convection currents, Eq. (1) is supplemented by the “material” relations:

$$\vec{D} = \varepsilon_c \vec{E}, \quad \vec{B} = \mu_c \vec{H}, \quad \vec{j} = \gamma_c \vec{E} + \gamma_c [\vec{u} \times \vec{B}], \quad (2)$$

where \vec{D} and \vec{B} are the electric and magnetic field induction vectors and γ_c is the specific electrical conductivity.

A complete set of equations with respect to the stress and strain tensor components and the displacement vector at given volumetric and surface forces will be written in the following way.

The equilibrium equations:

$$\sigma_{ij,j} + f_i = 0, \quad x_1, x_2, x_3 \in V, \quad (3)$$

where σ_{ij} are the stress tensor components and f_i are the components of the volumetric electromagnetic forces (more will be said below about their definition).

The Cauchy geometric relations for small strains:

$$\varepsilon_{ij} = 1/2 (u_{j,i} + u_{i,j}), \quad (4)$$

where ε_{ij} are the strain tensor components and u_i are the displacement vector components.

The system of equations (1)–(4) is supplemented by the boundary conditions:

$$\vec{E}_\Gamma \times \vec{n} = 0, \quad \vec{D}_\Gamma \cdot \vec{n} = 0, \quad \vec{H}_\Gamma \times \vec{n} = 0, \quad \vec{B}_\Gamma \cdot \vec{n} = 0, \quad (5)$$

$$\vec{\sigma}_n = \vec{p}_n + \frac{\Xi}{2} \vec{E}_\Gamma + \frac{\mu_c}{2} (\Xi \dot{\vec{u}}_\tau + \vec{i}) \times \vec{H}_\Gamma, \quad (6)$$

where $\vec{\sigma}_n$ is the stress vector on the surface with the normal \vec{n} , $\vec{\sigma}_n = \sigma \cdot \vec{n}$, Ξ and \vec{i} are the densities of surface charges and currents, and $\dot{\vec{u}}_\tau$ is the projection of the point's velocity vector on the plane tangential to the body boundary.

In the cases where it is necessary to consider the deformation of compound bodies for the points of contacting bodies, the matching conditions are derived for the vector characteristics of the EMFs and vector geometric characteristics of the deformation process:

$$\begin{aligned}(\vec{E}_1 - \vec{E}_2) \times \vec{n} &= 0, & (\vec{D}_1 - \vec{D}_2) \cdot \vec{n} &= 0, \\(\vec{H}_1 - \vec{H}_2) \times \vec{n} &= 0, & (\vec{B}_1 - \vec{B}_2) \cdot \vec{n} &= 0,\end{aligned}\quad (7)$$

$$u_n^{m-1} + u_n^{m+1} - \delta_{0n}^m \leq 0, \quad \sigma_{nn}^m \leq 0, \quad (8)$$

where u_n^{m-1} and u_n^{m+1} are the normal displacements of the points on the body surfaces, δ_{0n}^m is the initial gap (or interference), and σ_{nn}^m is the normal stress on the contacting surfaces. In the numerical solution of the problem, it is convenient to take into account the contact deformation by introducing the layers of contact elements [9, 10].

The generalized constitutive equations describing the relation between stresses and strains at the points of elastically deforming bodies can be represented by the tensor linear relationships between the stress and strain tensor components, ε_{ij} and σ_{ij} :

$$\varepsilon_{ij} = A_{ijkl} \sigma_{kl} + \alpha_{ij} \Delta T, \quad A_{ijkl}^e = \frac{1}{E} [(1 + \nu) \delta_{ik} \delta_{jl} - \nu \delta_{ij} \delta_{kl}], \quad (9)$$

where E and ν is the elastic modulus and Poisson's ratio, and A_{ijkl}^e and $\alpha_{ij} \Delta T$ are the components of the elasticity tensor and the material's thermal expansion.

According to the Prandtl-Reuss plasticity theory, we write the relationships between the strain and stress components in increments:

$$\left\{ \begin{aligned} d\varepsilon_{ij} &= A_{ijkl} d\sigma_{kl} + \phi_{ij} dT, \quad A_{ijkl} = A_{ijkl}^e + A_{ijkl}^p, \quad A_{ijkl}^p = \frac{3}{2\sigma_i} F_\sigma(\sigma_i, T) s_{ij} s_{kl}, \\ \phi_{ij} &= \phi_{ij}^e + \phi_{ij}^p, \quad \phi_{ij}^e = \delta_{ij} \frac{d\varepsilon^T}{dT} - \frac{1}{E^2} \frac{dE}{dT} [(1 + \nu) \sigma_{ij} - 3\nu \delta_{ij} \sigma_0] + \frac{1}{E} \frac{d\nu}{dT} (\sigma_{ij} - 3\delta_{ij} \sigma_0), \\ \phi_{ij}^p &= -F_\sigma(\sigma_i, T) \frac{\partial \sigma_T}{\partial T} s_{ij}, \quad F_\sigma(\sigma_i, T) = \frac{3}{2\sigma_i} \left(\frac{1}{E_k} - \frac{1}{E} \right), \end{aligned} \right. \quad (10)$$

where E_k is the tangent modulus.

The plastic flow law for isotropic material with a translational strain hardening is assumed to be associated with the condition of plasticity of the form:

$$\frac{3}{2} \left[s_{ij} - \frac{2}{3} \frac{EE_T}{E - E_T} (\varepsilon_{ij})_p \right] \times \left[s_{ij} - \frac{2}{3} \frac{EE_T}{E - E_T} (\varepsilon_{ij})_p \right] - \sigma_Y^2 = 0, \quad (11)$$

where s_{ij} is the stress deviator, E_T is the hardening modulus, σ_Y is the material yield strength, $\sigma_0 = 1/3 \sigma_{kl} \delta_{kl}$, $\varepsilon_0 = 1/3 \varepsilon_{kl} \delta_{kl}$, and σ_i is the stress intensity as a function of the strain intensity and temperature, $\sigma_i = H(\int d\varepsilon_i^p, T)$.

One of the most efficient techniques for solving the posed problem is the finite element method. The basis for its specific application can be the method of variable elastic parameters, together with the step-by-step solution of the equilibrium state problem using the variational method, which reduces the solution of the posed problem to the variational equality

$$\delta E^{k+1} = 0, \quad E^{k+1} = U^k + W, \quad (12)$$

where U^k is the energy of “elastic” deformation, which is calculated from the SSS in the previous k th step of determining the elastic parameters, and W is the energy of the EMF which is calculated from the values of the EMF vectors in the k th step. In case of the neglect of the influence of the electric field, the energy of the EMF can be determined only in terms of the magnetic field parameters:

$$W = \int_V \left(\int_0^B \vec{H} d\vec{B} \right) dV. \quad (13)$$

As a first approximation, we can neglect the variability of the material magnetic permeability and consider it constant. Then, we introduce into consideration the so-called vector magnetic potential \vec{A} , which is related to the magnetic induction vector in the following way:

$$\vec{B} = \text{rot } \vec{A}. \quad (14)$$

The EMF energy can be written in the matrix-vector form:

$$W = \frac{1}{2} \{A\}^T [M] \{A\}, \quad (15)$$

where $[M]$ is the matrix of the material’s “magnetic properties,” the elements of which are dependent on the material’s properties, body geometry and are determined using the Maxwell fundamental equations (1) in view of relationship (14).

The energy of elastic deformation can be represented by a similar matrix-vector equation:

$$U^k = \frac{1}{2} \{u\}^T [K]^k \{u\}, \quad (16)$$

where $\{u\}$ is the column-vector of nodal displacements and $[K]^k$ is the stiffness matrix.

Then, variational equation (12) in view of (15) and (16) is written as

$$\delta E^{k+1} = 0 \Rightarrow \left\{ \begin{array}{l} \frac{\partial E^{k+1}}{\partial A} = 0 \\ \frac{\partial E^{k+1}}{\partial u} = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} [M] \{A\} + [M_k] \{u\} = 0, \\ [K]^k \{u\} + [K_m] \{A\} = 0, \end{array} \right. \quad (17)$$

$$[M_k] = \frac{1}{2} \{u\}^T \frac{\partial [K]}{\partial \{A\}}, \quad [K_m] = \frac{1}{2} \{A\}^T \frac{\partial [M]}{\partial \{u\}}.$$

The matrices $[M_k]$ and $[K_m]$ govern the influence of the variation in the EMF components on the mechanical properties of the material. Moreover, they define the relationship between the EMF and mechanical fields. With the neglect of the magnetostriction effects, the problems of determining the EMF and SSS components can be considered separately from one another. Here, the product $[K_m] \{A\}$ with an opposite sign has the meaning of the column-vector of nodal electromagnetic forces. Indeed, if the principle of virtual work is used to determine the forces acting on the points of the deformed electrically conductive body in the presence of EMF, we obtain

$$F_{em} = -\frac{\partial W}{\partial u} = -\frac{\partial}{\partial u} \left(\int_V \left(\int_0^B \vec{H} d\vec{B} \right) dV \right) = -\int_V \vec{H} \frac{\partial \vec{B}}{\partial u} dV - \int_V \vec{H} \frac{\partial V}{\partial u} d\vec{B}. \quad (18)$$

If the influence of the electric field and the variation in the material magnetic permeability are ignored, then, considering the matrix-vector notations we have

$$\{F_{em}\} = -\frac{\partial W}{\partial u} = -\frac{\partial}{\partial u} \left(\frac{1}{2} \{A\}^T [M] \{A\} \right) = -\frac{1}{2} \{A\}^T \frac{\partial [M]}{\partial u} \{A\} = -[K_m] \{A\}. \quad (19)$$

Thus, in this case, the problem of determining the EMF vector characteristics and deformation tensor characteristics is reduced to the system of two equations, which can be solved in series:

$$\begin{cases} [M] \{A\} = 0; \\ [K] \{u\} = \{F_{em}\}, \quad \{F_{em}\} = -[K_m] \{A\}. \end{cases} \quad (20)$$

Note that this approach to the determination of electromagnetic forces is found elsewhere, e.g., in [11]. The reduced system of equations (20) is supplemented by the conditions for the vector magnetic potential and displacements on the body surfaces. The electric current, which is required to flow in a closed loop of one or several bodies of the system, can be considered as the EMF source.

Example of Calculation. Consider the application of the proposed method to the analysis of the coil–workpiece system deformation using the technology of PMTM, the purpose of which is to smooth out dents in thin sheet workpieces. The conceptual possibilities for performing this technological step are presented in detail in [12–15], where it is proposed to use a large, bandaged single-turn coil as the source (Fig. 1). Due to special features of the coil geometry, the generated electric current is concentrated in the area of the conical working aperture, as confirmed by the experimental observations. This kind of coils can be successfully used for the attraction of a metal having a relative magnetic permeability greater than unity.

The studies of [16–18] report the results of application of the proposed method for numerical determination of the EMF distribution and strain state of the coil–workpiece system for simplified computational models similar to those for which analytical solutions were obtained [12–14].

The comparison of the numerical and analytical results with the data of experimental investigations on determination of the EMF components is indicative of the possibility of using the proposed method in the analysis of EMF component distribution in complex technological systems of PMTM. Besides, the SSS analysis of the unbandaged coil–workpiece system was made, showing that the stress tensor components increase significantly with an increase of the current intensity in the coil, which can result in its irreversible deformation and fracture. This fact was substantiated by testing bandaged prototypes wherein bandages were used to improve the strength properties.

Consider the computational model for the bandaged coil–workpiece system according to the axisymmetric formulation of the problem (Fig. 2a). The solution was performed with the following geometrical parameters of the system: the outer diameter of the coil is 100 mm; the coil thickness is 15 mm; the angle of inclination of the conical surface is 60°; the outer diameter of the bandage is 130 mm; the bandage thickness is 45 mm; the plate (workpiece) thickness is 2 mm. Also investigated was the deformation of the plate with a dent, whose diameter was chosen equal to, greater and less than that of the working aperture of the coil (Fig. 2b, c, d). The physico-mechanical properties of the computational model elements were taken as follows: coil (the material is copper): the relative magnetic permeability $\mu_r = 1$, elastic modulus $E = 180$ GPa, Poisson's ratio $\nu = 0.33$, yield strength $\sigma_Y = 200$ –210 MPa; bandage (the material is fiberglass): the relative magnetic permeability $\mu_r = 1$, elastic modulus $E = 35$ GPa, Poisson's ratio $\nu = 0.35$; workpiece (the material is steel): the relative magnetic permeability $\mu_r = 1.3$, elastic modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.28$, yield strength $\sigma_Y \sim 300$ –380 MPa; air: the relative magnetic permeability $\mu_r = 1$. The problem was solved in the quasi-stationary formulation, the current uniformly distributed along the line Γ_2 ($I_m = 50$ kA) was considered as the field source (Fig. 2a).

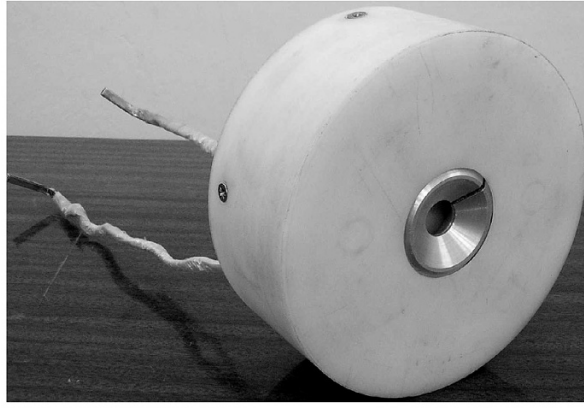


Fig. 1. Coil system for attracting thin sheet workpieces.

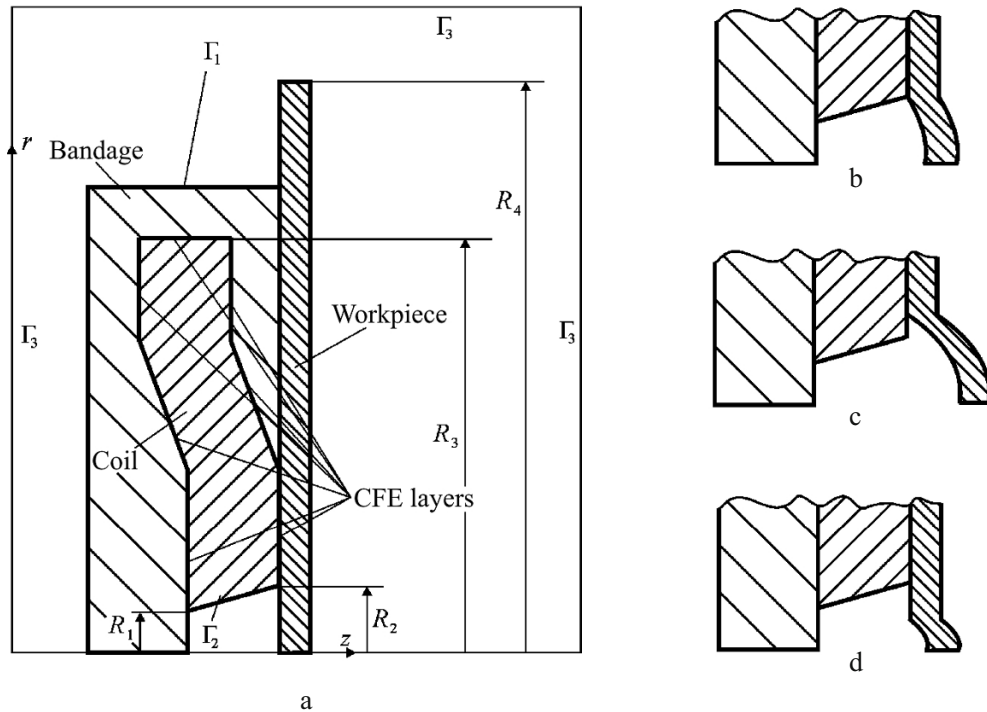


Fig. 2. Computational model for the bandaged coil-workpiece system.

The layers of contact finite elements (CFE) integrating separate elements into a unified system were introduced in the model between the coil, bandage, and workpiece. Besides, using the CFE, the interference fit between the bandage and coil was simulated. The attaching conditions ($u_r = u_z = 0$) were specified on the line Γ_1 (Fig. 2a). The distribution of the EMF vector components was obtained at the first stage of the problem solution, with the conditions for the EMF decay at a distance from the source being satisfied, which is equivalent to $A|_{\Gamma_3} = 0$.

The deformation of the system was analyzed at the second stage. The main results of the analysis are as follows. The maximum values of the stress intensity in the coil are observed in close proximity to the working aperture; those in the workpiece are observed opposite to the working aperture, with the stress intensity value in the workpiece material being above the yield strength. Figure 3 illustrates the stress intensity distribution in close proximity to the working aperture and the strain state of the system.

The comparison of the results obtained for the quantity having different diameters shows that in the case where the diameter is less than the working aperture diameter, the technology under consideration fails.

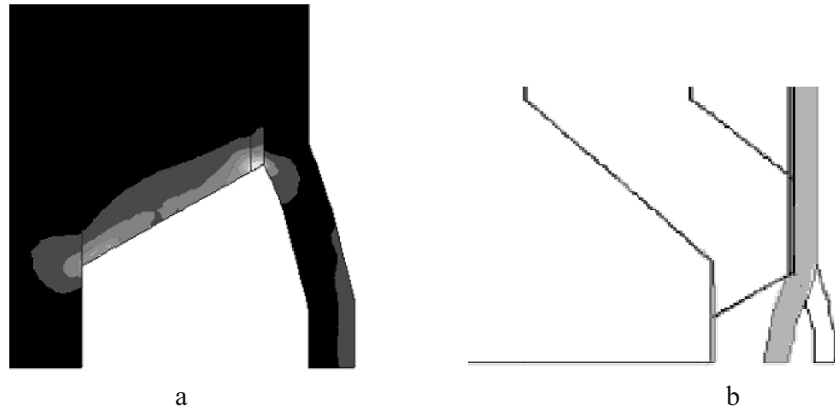


Fig. 3. Stress-strain state of the coil, bandage, and workpiece (with a dent): (a) distribution of the stress intensity in close proximity to the working aperture; (b) strain state of the system. (Black-out part is deformed workpiece.)

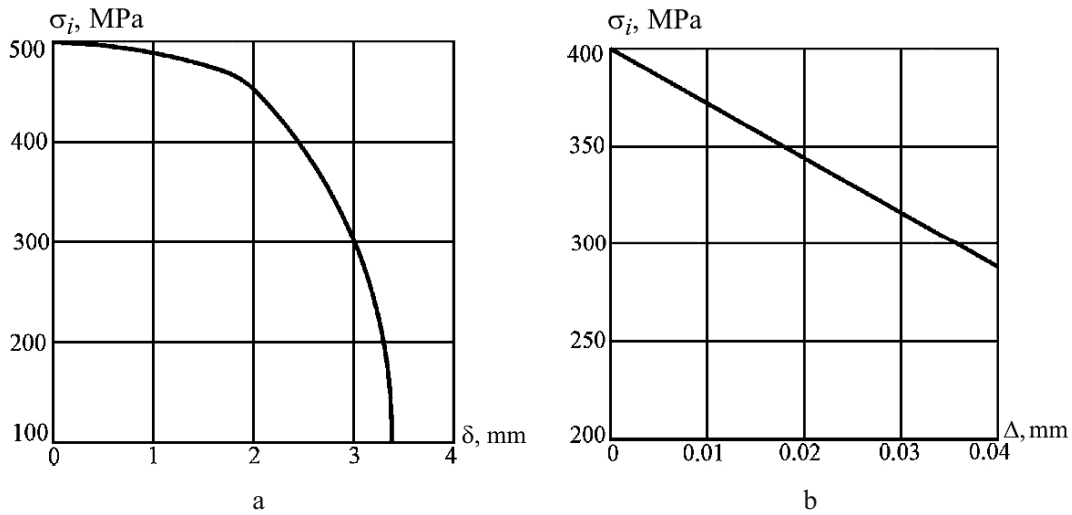


Fig. 4. The maximum stress intensities in the matrix as a function of the dent depth δ (a) and in the coil as a function of the bandage interference fit Δ (b).

In addition, the calculations were performed, in which the dent depth (Fig. 2a) and the interference fit between the bandage and coil (Fig. 4) were varied for a given value of the current intensity. It is seen from Fig. 4 that for the dent depth higher than 3 mm, the yield strength in the workpiece material for a given value of the current intensity is not reached. With an increase in the interference fit of the bandage, the stresses in the coil decrease considerably. The comparison of the obtained data with those given in [17, 18] shows that the stress intensity in the coil strongly depends on the interference fit value. Thus, the use of bandages for this type of coils allows the increase in the current intensity of the pulse with no significant stress increase in the coil.

CONCLUSIONS

1. A mathematical formulation and method for the analysis of elastoplastic deformation in the system of electrically conductive bodies under the action of electromagnetic field are developed. The numerical implementation of the method is based on the finite element method and the method of variable elastic parameters together with the step-by-step solution of the equilibrium state problem.

2. With the pulsed magnetic treatment of materials aimed at straightening dents on thin-wall structural elements, the computational model has been developed for bodies interacting through contact that involves an

inductor, or coil, bandage and workpiece, and the stress-strain state analysis has been performed, which made it possible for the first time to assess the short-term strength characteristics of the coil.

3. The influence of the manufacturing and design parameters on the distribution of the stress-strain state components in the coil and workpiece is analyzed. It has been found that the use of the bandage connected with interference fit to the coil allows a significant reduction in the stress intensity value of the coil. The degree of size influence of the dent in the workpiece under treatment on its stress-strain state has been established as well.

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